# NASA TECH BRIEF



## Computer Program Provides Linear Sampled-Data Analysis for High Order Systems

### The problem:

To design a computer program to accommodate S-plane transfer functions of systems up to a limit of 50th order and to provide frequency response, step and ramp response, and root locus trajectories in either the W-plane or Z-plane, as practicable. The program should also feature a direct S-to-W transformation, followed by a W-to-Z transformation.

It was found that standard practices in the analysis of linear sampled-data control systems were not suited to high order systems. The problem appeared to be associated with factoring Z-plane polynomials to get Z-plane poles and zeros. Prior practice was to perform transformations in the order S-to-Z-to-W.

### The solution:

A program designed to perform transformations in the order S-to-W-to-Z which allows certain arithmetic to be completed in the W-plane, rather than in the Z-plane where computer resolution degrades the arithmetic. The new method is based on a direct transformation from the S-plane to the W-plane. The arithmetic required to get the W-plane poles and zeros is not penalized noticeably by digital computer resolution, and great accuracy is achieved. The W-plane poles and zeros are then quite easily transformed into Z-plane poles and zeros using the well known bilinear transformation algorithm.

### How it's done:

Partial Fraction Expansion: uses one method of simultaneous equations to calculate the constants (partial fraction coefficients) in the partial fraction series.

S-to-W Transformation: uses the method of algorithms to calculate W-plane substitutes for the S-plane terms of the partial fraction series, and the method of polynomial root extraction to obtain W-plane zeros. The algorithms give the W-plane poles directly.

W-plane Root Locus: manipulates the W-plane numerator and denominator polynomials to generate the characteristic polynomial equations and uses the method of polynomial root extraction to obtain the roots.

W-plane Frequency Response: evaluates the W-plane numerator and denominator polynomials for real and imaginary parts for purely imaginary values of the argument.

W-to-Z Trasnformation: uses the method of bilinear transformation, for each W-plane pole and zero.

### Notes:

1. This program is written in Fortran IV for an IBM 7094 computer.
2. This program can be used to obtain frequency response, step and ramp response, and root locus calculations of linear single rate sample data systems up to a recommended limit of 50th order. The complex W- and Z-planes are employed in the calculations. For practical reasons root locus calculations are performed only in the W-plane, and step/ramp response calculations are performed only in the Z-plane.

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3. The program features an S-to-W plane transformation, automatic data hold selection, and provision to enter real time delays as part of the input S-plane transfer function. The program also features a W-to-Z plane transformation, and provisions for entering W-plane and Z-plane terms directly in place of, or in addition to, S-plane data inputs. An automatic loop closure option is also provided for unity feedback gain systems. This allows closed loop step and/or ramp response calculations to be obtained from open loop input data. Closed loop frequency response is also made possible by this feature.

4. Inquiries concerning this program may be directed to:

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   Reference: B67-10287

**Patent status:**

No patent action is contemplated by NASA.

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